

Unit 1: Limits and Derivatives

AP/ECE Calculus

8 Class Meetings

Revised November 2019

Essential Questions

- What is the relationship between limits and derivatives?

Enduring Understandings with Unit Goals

EU 1: A function's continuity at a given point affects its limit

- Explain how to use the Precise Definition of a Limit
- Evaluate a limit for functions with discontinuities
- Use the formal definition of continuity to explain whether or not a function is continuous at that point

EU 2: Derivatives can be used to analyze curves and model rates of change.

- Use the difference quotient to find a derivative at a point
- Apply the concept of derivative to write tangent line equations
- Identify how a function is changing based on its derivative

EU 3: Differentiability implies continuity.

- Identify points where a function may be continuous and not differentiable.

EU 4: Finding the derivative of a function may require the use of several rules, including rules for: sums, products, quotients, powers

- Evaluate a derivative using the rules

EU 5: The physics concepts of position, velocity and acceleration are related mathematically by the derivative

- Find the instantaneous rate of change of a function at a given point.
- Determine an object's displacement, distance traveled, and speed based on a graph or a given function.

Standards

AP Calculus Curricular Requirements – College Board

- **CR1a** The course is structured around the enduring understandings within Big Idea 2: Derivatives.
- **CR2a** The course provides opportunities for students to reason with definitions and theorems.

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- **CR2b** The course provides opportunities for students to connect concepts and processes.
- **CR2c** The course provides opportunities for students to implement algebraic/computational processes.
- **CR2d** The course provides opportunities for students to engage with graphical, numerical, analytical, and verbal representations and demonstrate connections among them.
- **CR2e** The course provides opportunities for students to build notational fluency.
- **CR2f** The course provides opportunities for students to communicate mathematical ideas in words, both orally and in writing.
- **CR3a** Students have access to graphing calculators.
- **CR3b** Students have opportunities to use calculators to solve problems.
- **CR3c** Students have opportunities to use a graphing calculator to explore and interpret calculus concepts.
- **CR4** Students and teachers have access to a college-level calculus textbook.

MSMHS Academic, Civic, and Social Competencies

Competency 1. Read and write effectively for a variety of purposes

Competency 2. Speak effectively with a variety of audiences in an accountable manner

Competency 3: Make decisions and solve problems independently and collaboratively.

Competency 5. Contribute to a positive learning environment with respect and responsibility.

Unit Content Overview

1. Precise Definition of a Limit

- Identify the key components of the definition
- Evaluating limits graphically using the definition

2. Continuity

- Identify different types of continuity
- Explain the three requirements for establishing continuity
- Apply the Intermediate Value Theorem for continuous functions

3. Derivative of a Function

- Find a derivative at a given point using the definition of derivative
- Match the graph of a function with the graph of its derivative
- Write an equation of a tangent line to a curve at a given point

4. Differentiability

- Use graphs to analyze differentiability and continuity
- Find a numerical derivative algebraically and on a calculator
- Utilize end behavior models to evaluate limits involving infinity

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5. Rules for Differentiation

- Apply the shortcut (power rule) for finding a derivative
- Find the derivatives of the six trig functions, exponential and log functions and inverse trig functions
- Find the derivative using logarithmic differentiation
- Evaluate a derivative using the product and quotient rules
- Find higher order derivatives

6. Velocity and Other Rates of Change

- Find instantaneous and average rates of change of area and volume formulas
- Determine when a particle moving along a line is speeding up, slowing down and changing direction

Interdisciplinary Connections

- Physics – Free fall and speed problems

Daily Learning Objectives with *TWPS Activities*

Students will be able to...

- Explain how to use the Precise Definition of a Limit
 - *How does the Precise Definition of a Limit relate to the definition we learned in the summer unit (Definition of a Limit)?*
- Articulate the three necessary items for determining continuity
- Apply the Intermediate Value Theorem for Continuous Functions
- Write the equation of a tangent line at a given point
- Use the definition of derivative to analyze a function
- Use the right and left-hand derivatives to show whether or not a function is differentiable and/or continuous
- Apply the rules of differentiation to find derivatives of functions
- Analyze the graph of a position, velocity and acceleration function

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Instructional Strategies/Differentiated Instruction

- **HLP:** Academically Productive Talk
- **HLP:** Writing to Learn (TWPS)
- **HLP:** Effective Feedback
- Daily TWPS AP/ECE Warm-ups
- Lecture with notes
- Guided notes
- Accountable Talk
- Student-led instruction
- Independent problem-solving
- Collaborative problem-solving
- Cross-curricular problem solving (independent and collaborative)
- Homework

Assessments

FORMATIVE ASSESSMENTS:

- White board examples
- Mid-class check-ins
- Student explanations of continuity and differentiability
- Exit Slips
- Homework
- Accountable Talk Discussions
- Daily Think-Write-Pair Share (TWPS)
- AP Exam Prep questions
- Matching with Derivatives Performance Task
 - MSMHS Rubric 3: Problem Solving

SUMMATIVE ASSESSMENTS:

- Quiz on EU 1, 2 and 3
- Unit Test
- Matching with Derivatives Performance Task

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Unit Task

Unit Task Name: Continuity and Velocity FRQ

Description: Students will use information learned in this unit about how a functions' continuity affects its limit (EU 1) how to use derivatives to analyze curves (EU 2), how differentiability implies continuity (EU 3) some functions require rules to find their derivatives (EU 4) how velocity, position and accelerations are related through derivatives (EU 5) in order to match function graphs with their derivative graphs. Students will be given a group of 16 function graphs and expected to match them to their derivative graphs. They will be required to write at least two sentences for each function justifying why they chose the particular derivative.

Evaluation: MSMHS Rubric 3: Problem Solving

Unit Resources

- Finney, Ross L., Demana, Franklin D., Waits, Bert K., Kennedy, Daniel. *Calculus: Graphical, Numerical, Algebraic*. Fourth ed., Prentice Hall, 2012.
- Stewart, James. *Single Variable Calculus: Early Transcendentals*. Cengage Learning, 2016.
- MSMHS School-wide Rubrics
- Internet databases
- Laptops
- Graphing Calculators